Proxy Modeling

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Agenda

1. Introduction to Modeling Efficiency

2. Proxy Modeling

- Curve Fitting
- “Curve Fitting+”: Least Squares Monte Carlo

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Introduction to Modeling Efficiency

Examples of modeling efficiency

- **Modeling techniques**
  - Traditional actuarial model building (cell compression) – Places assets and liabilities into “like” categories/buckets, and these buckets then used for valuation
  - Cluster modeling – More advanced form of cell compression
  - Scenario reduction – Pick a subset of scenarios from a “full” population that still adequately captures all the risk exposures
  - Replicating portfolio – Find a basket of assets that mirrors the liabilities and then use this basket of assets for valuation work (usually more manageable than working with liability models)
  - Proxy modeling – Fit a function (proxy) to the assets and/or liabilities and use the function for valuation work. Proving to be very effective for “nested stochastic.”
- **Technology**
  - Grid computing
  - Cloud computing
  - GPGPU (General Purpose Graphics Processing Unit) processors

Why is modeling efficiency becoming increasingly important?

- **Small companies**
  - Smaller companies likely not to have any VA business so up to this point likely to have had no requirement for stochastic modeling
  - Stochastic modeling required under VM 20 is going to have a significant impact
  - Some may already be up against systems issues just doing cash-flow testing
- **Hedge effectiveness testing**
  - In order to get credit for hedging in reserves/capital under AG43/C3 Phase II need to project hedges under a variety of stresses, and ideally do it on a fully stochastic basis. Hence a "nested stochastic" or "stochastic-on-stochastic" computation, which will be very onerous if done on a "brute force" basis.
- **ORSA**
  - Projection of stat or economic capital again potentially means a nested stochastic calculation
- **Non-regulatory applications also create heavy computational demands**
  - Internal EC and ALM
  - Product pricing where stat reserves/capital need to be projected

AAA MEWG

- The regulators and LFS/RM Committee recognized that some of the calculations envisioned by a PBA approach to reserves and capital can be onerous
- The purpose of the Modeling Efficiency Work Group (MEWG) is to provide information to the regulators and LFS/RM, and valuation actuaries on ways in which these calculations can be made more "efficient"
  - By efficient we mean ways in which modeling techniques and/or technology can be used to help ease the burden of huge processing requirements without affecting the accuracy of the results
  - For example, a scenario reduction technique is only "efficient" if it leads to the same result as a calculation using a larger scenario set
- Our focus is on providing information and data from which interested parties can make informed decisions. We categorically do not attempt to favor any one tool or technique over another.
  - In the spirit of true PBA, and given some techniques may be more appropriate than others depending on the application/metric you are looking at, we believe prescribing anything in this area would be inappropriate.

What has MEWG achieved?

- Created bibliography of published materials regarding modeling efficiency
- Two industry surveys:
  - "Modeling Efficiency Work Group Survey – November 2007"
    - 29 participant companies.
    - 80% using model building techniques; 40% using "scenario design"
    - 25% using "hardware design;" 30% "software design"
  - "Usage of Modeling Efficiency Techniques in the US Life Insurance Industry" – April 2013
    - 51 participants
    - 25% using cell compression outside of traditional actuarial mapping
    - 46% of participants now indicate using scenario reduction techniques (including the Academy scenario picking tool)
    - Proxy approaches gaining popularity
- MEWG provided input and guidance on drafting "modeling efficiency" sections of VM 20
- MEWG members have also been very active in writing papers and presenting at conferences on modeling efficiency topics

A new era for MEWG

- MEWG presented at Aug 22 LATF meeting, and coming out of that meeting, regulators emphasized two key areas they need help with going forward:
  - Continued ad hoc advice and support from time-to-time, as issues arise, similar to what we did for VM 20
  - More pressing and difficult area: How do they assess any particular technique that is being used by a company they are reviewing?
- Regulator assessment
  - Resources
    - Do AAA working groups and/or the regulators need to engage quant (non-actuarial) professionals?
  - Understanding
    - Glossary of terms
    - What are key metrics for analysis?
    - Update recent SOA survey of techniques

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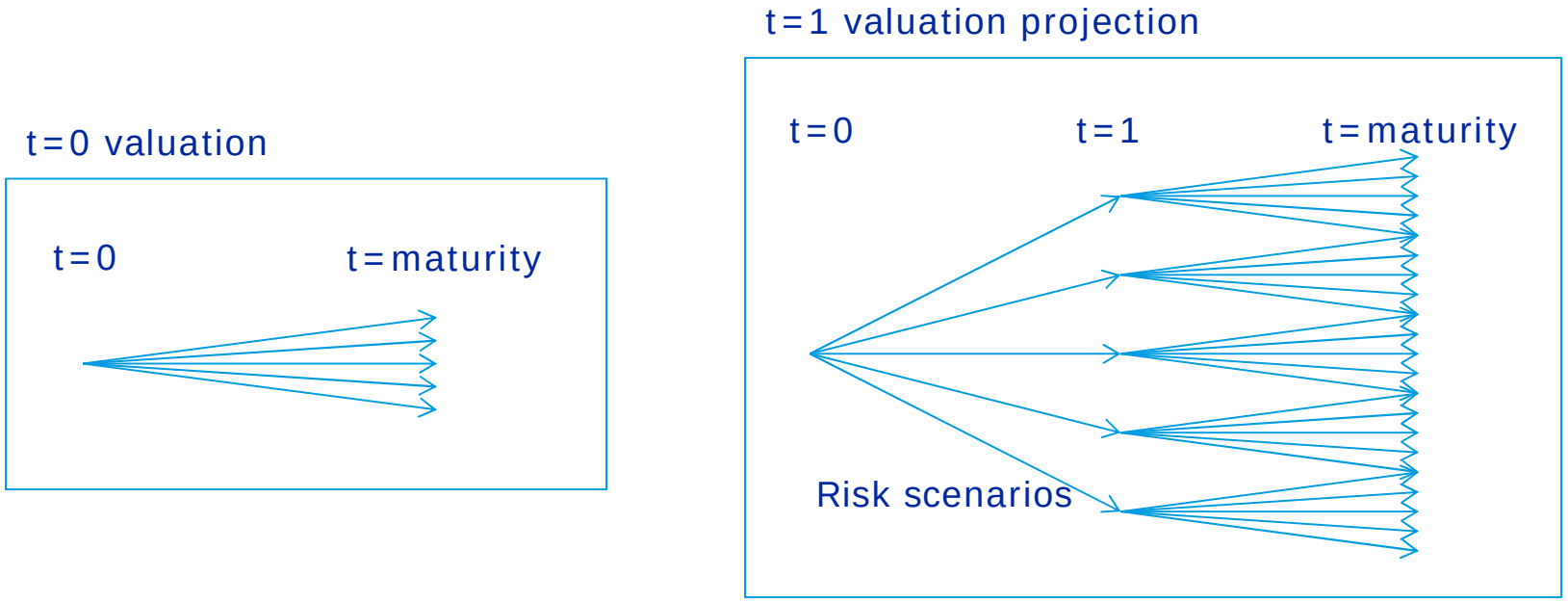
Proxy Modeling

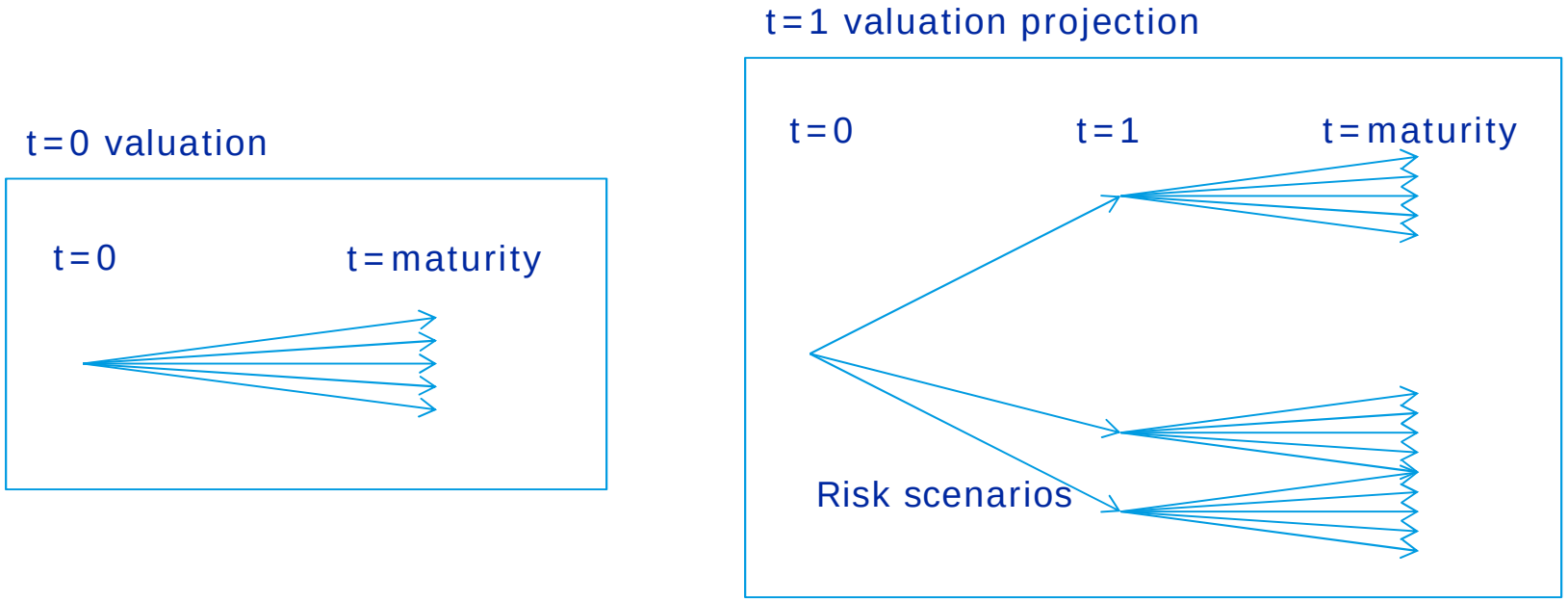
The Nested Stochastic Problem

- Complex liabilities such as VA living benefits may require a Monte Carlo simulation for valuation
- This introduces a 'Nested Stochastic' problem for some applications, e.g., EC calculated on a 1-year VaR MC basis
  - So perhaps 1,000 outer scenarios x 1,000 inner scenarios = 1 million scenarios



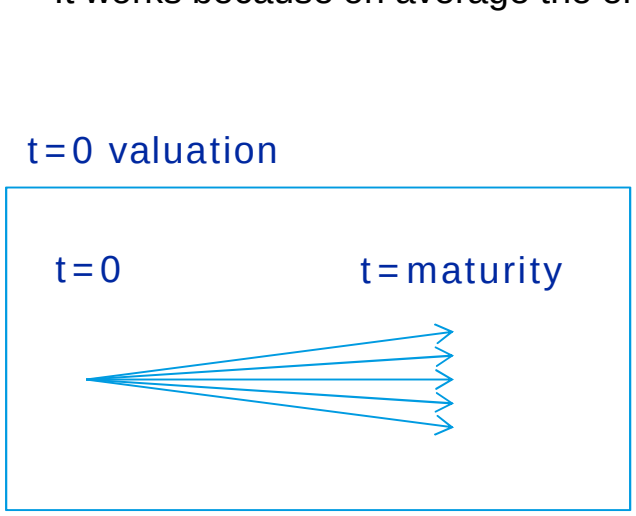
Proxy Approach: Curve Fitting

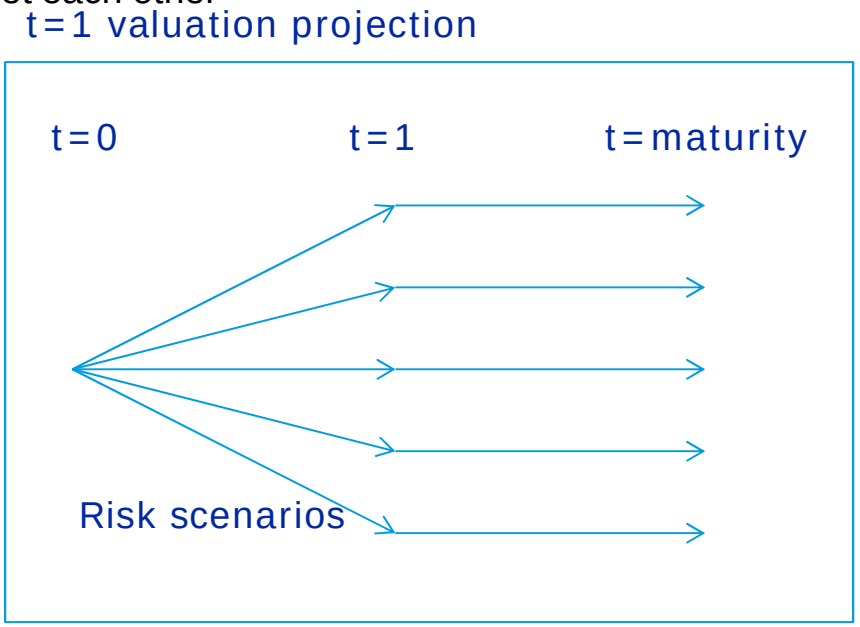
- Curve Fitting is a general technique to make Nested Stochastic problems feasible
- Rather than run many Real-World or Risk Scenarios, we run a small number, then perform a regression of these values against a number of key risk drivers.
  - Produces a proxy function
  - May enable us to get our 1 million scenarios down to 100 outer x 1,000 inner = 100,000 scenarios
- Issue: curve only good for the points that we've fitted to



Proxy Approach 2: Least Squares Monte Carlo (LSMC)

- LSMC = “Curve Fitting+”
- Rather than run many valuation scenarios for each Risk Scenario, we run a small number, then perform a regression of these values
  - Produces a proxy function
  - May enable us to get our 1 million scenarios down to 20,000 outer x 1 inner = 20,000 scenarios
  - Permits us to fit to many points, as opposed to the handful we have for Curve Fitting, hence we get a curve that works across the entire risk spectrum
  - It works because on average the errors offset each other



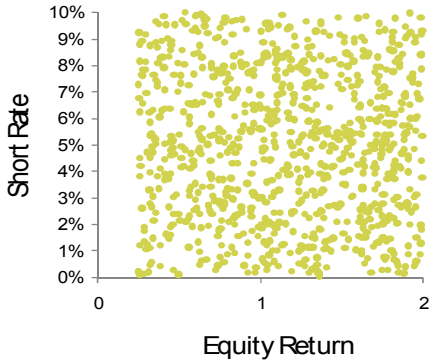


Process for Proxy Generation using LSMC

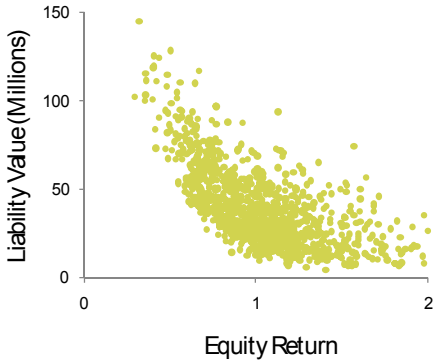
There are four main steps followed to derive the proxy function

Identify risks and generate 'fitting points'

Step 1



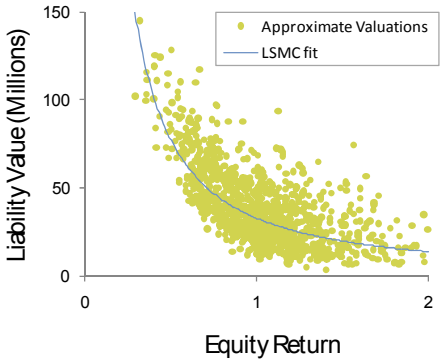
Step 2



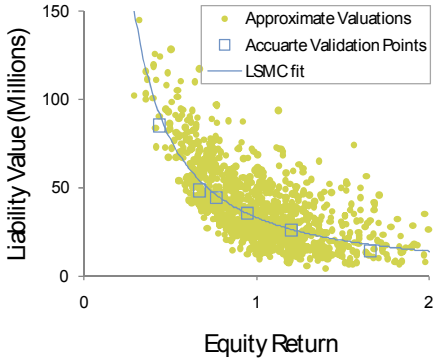
Calculate PV for each fitting point

Run optimization to fit PVs

Step 3

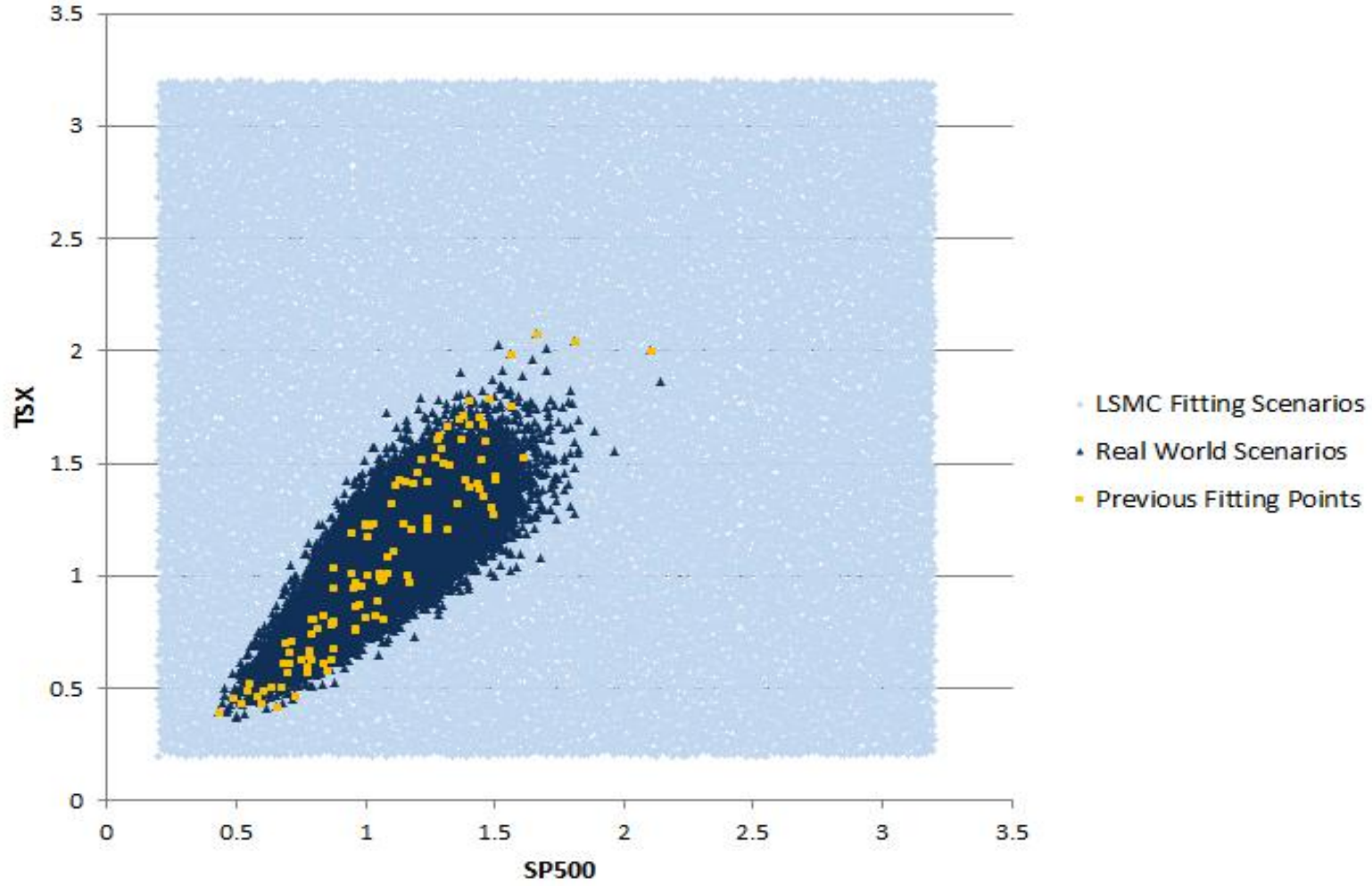


Step 4

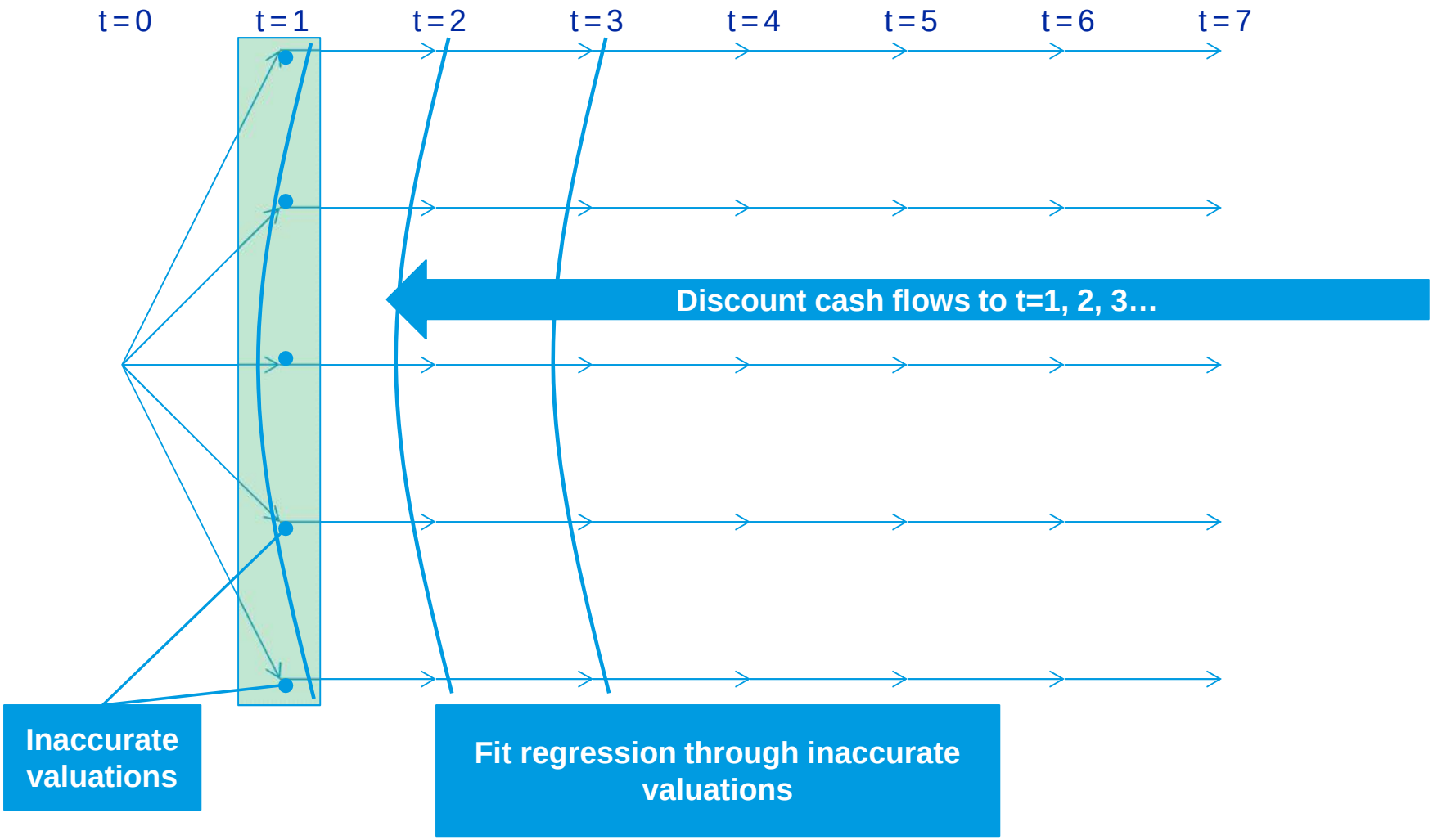


Validate proxy function using accurate valuations

Model Comparison – LSMC actually more accurate than Brute Force



Multi time-step LSMC – fitting scenarios



LSMC Example: Variable Annuities

Choosing risk drivers

Balance risk coverage with dimensionality of problem and risk budget

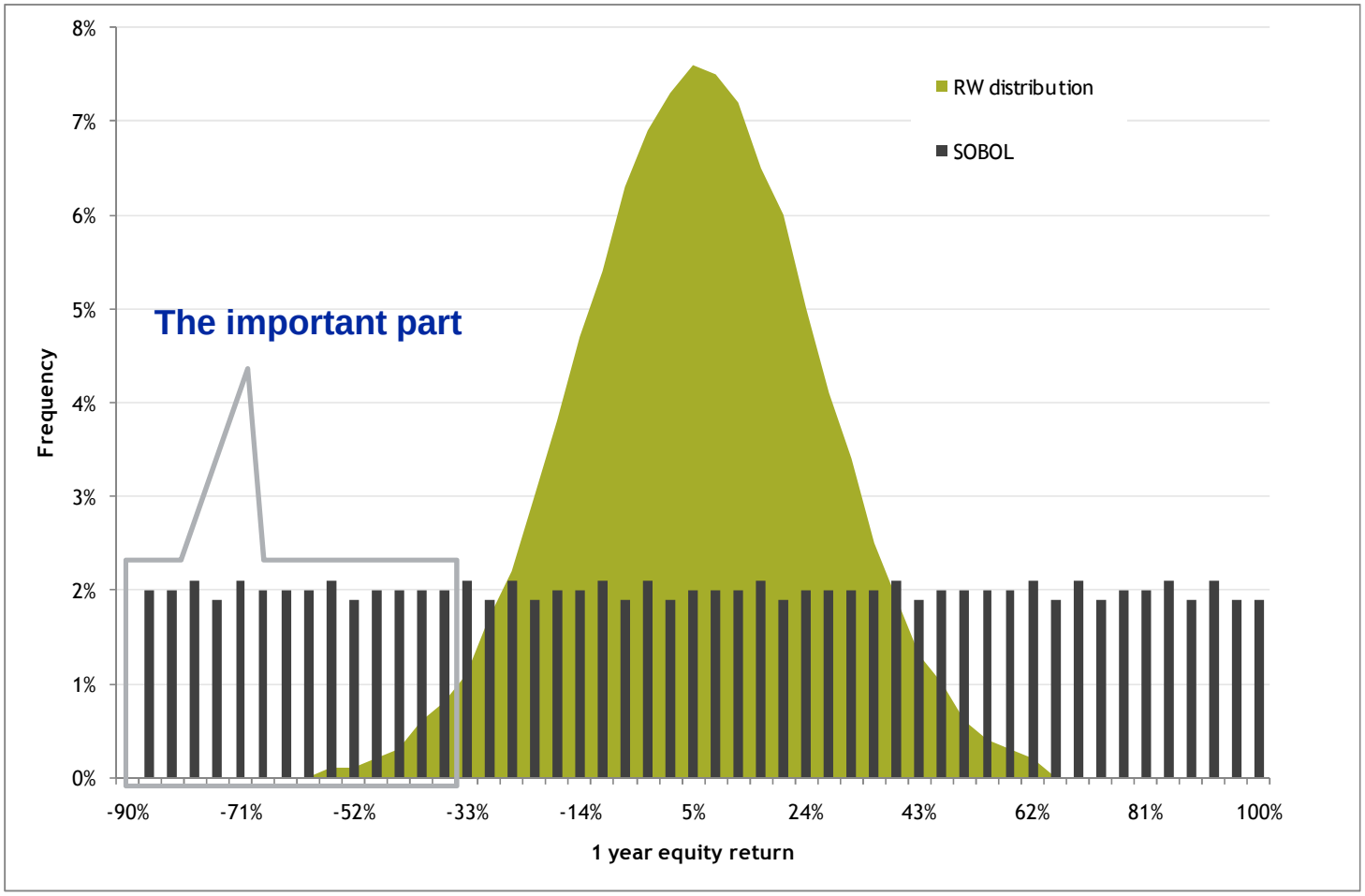
Example: VA block of business

- **Treasury curve level**: parameterized by two principal components, PC1 and PC2.
- **Equity level**: the return of the S&P500 index over 1 year
- **Equity volatility**: the option implied volatility of the S&P500 in 1 year
- **Swap spread**: the difference between the Treasury and Swap curves in 1 year

Key consideration: fully spanning risk driver space

- Interested in the rare, extreme events
- But Real World risk factor distributions rarely contain rare events, by definition
- Instead we create specialist “fitting” scenarios

Generating Fitting Scenarios



Proxy Function

For the VA book we generated a polynomial with 47 terms:

» P[,] are Legendre Polynomials e.g. P[4,2] is 2nd order Legendre in 4th risk driver

Term	Coefficient
(Intercept)	-992032984.7
P[1,1]	761762317.8
P[4,1]	1825715949
P[5,1]	-1395382145
P[4,2]	-758349090.5
P[3,1]	573661632.5
P[2,1]	550827457.8
P[5,2]	-241427978.9
P[4,3]	149133865.5
P[5,3]	75635590.08
P[3,2]	-62050937.12
P[4,4]	56961766.44
P[1,2]	-37964304.99
P[2,2]	-15170800.63
P[4,5]	-22140611.05
P[4,1]:P[5,1]	-696164796.1
P[1,1]:P[5,1]	385012027.6
P[1,1]:P[3,1]	-289866012.2
P[5,1]:P[2,1]	284099296
P[5,1]:P[3,1]	263166778.5
P[3,1]:P[2,1]	-209026930.8
P[1,1]:P[4,2]	203983067.4
P[5,1]:P[4,2]	192324722.4
P[1,1]:P[4,1]	-147402170

Term (cont)	Coefficient (cont)
P[4,1]:P[3,1]	-136432879.5
P[4,2]:P[2,1]	151225297.4
P[4,1]:P[5,2]	-148168834.3
P[4,1]:P[2,1]	-106150736.7
P[4,2]:P[3,1]	118719778.8
P[1,1]:P[2,1]	-60328828.74
P[1,1]:P[5,2]	50714322.09
P[2,1]:P[1,2]	-35340421.37
P[4,1]:P[1,2]	-34754483.49
P[5,1]:P[1,2]	-33940634.71
P[3,1]:P[5,2]	31296183.31
P[3,1]:P[1,2]	-29170013.58
P[2,1]:P[5,2]	27967168.13
P[1,1]:P[2,2]	-38769610.03
P[5,1]:P[2,2]	28575088.78
P[1,1]:P[4,1]:P[5,1]	411342943.2
P[4,1]:P[5,1]:P[3,1]	236861499.7
P[4,1]:P[5,1]:P[2,1]	265609661.1
P[1,1]:P[5,1]:P[2,1]	64154799.17
P[1,1]:P[4,1]:P[2,1]	-64631715.07
P[1,1]:P[5,1]:P[3,1]	-60439171.12
P[5,1]:P[3,1]:P[2,1]	-37145207.65
P[1,1]:P[4,1]:P[3,1]	-33812469.43

	Risk Driver
1	PC1
2	PC2
3	Swap Spread
4	Equity Value
5	Equity Vol.

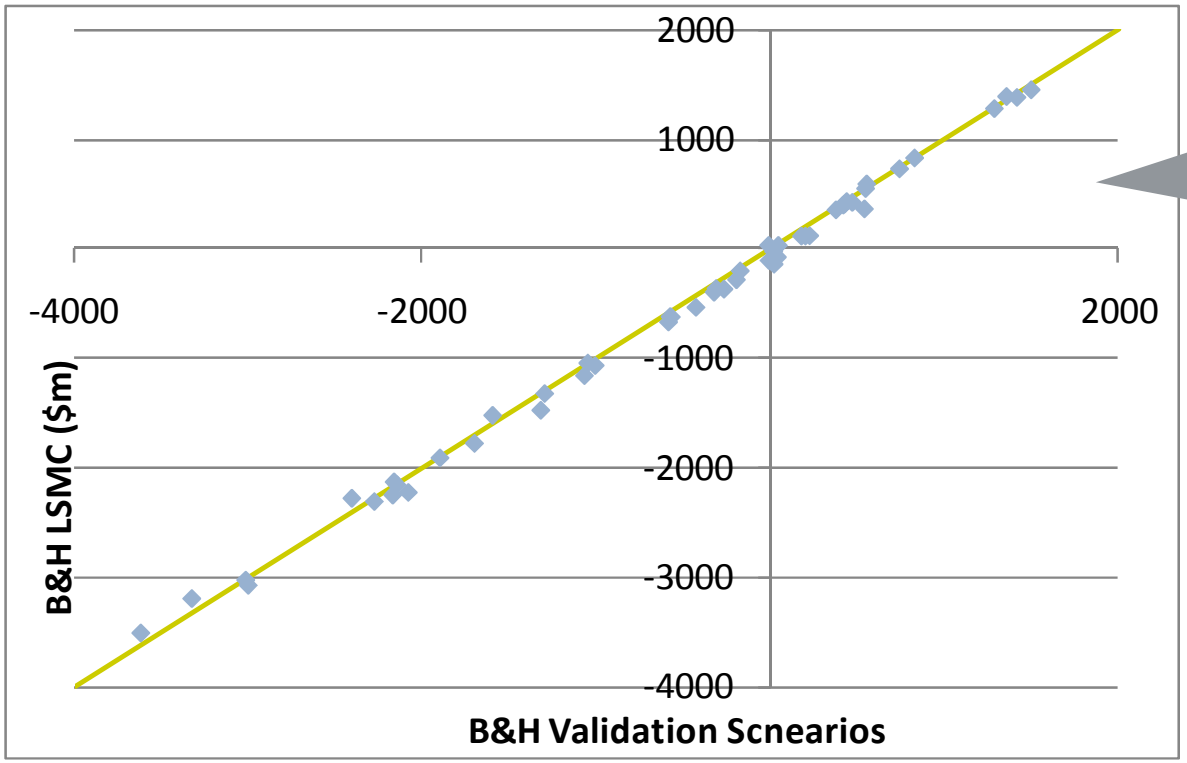
Up to 5th order in equity value term

Not all possible cross terms -> not overfitting

Validation

Comparison of 1-year surplus calculated using the proxy and calculated using 1,000 RN scenarios

» The average difference between the validation and the proxy function is $60 million



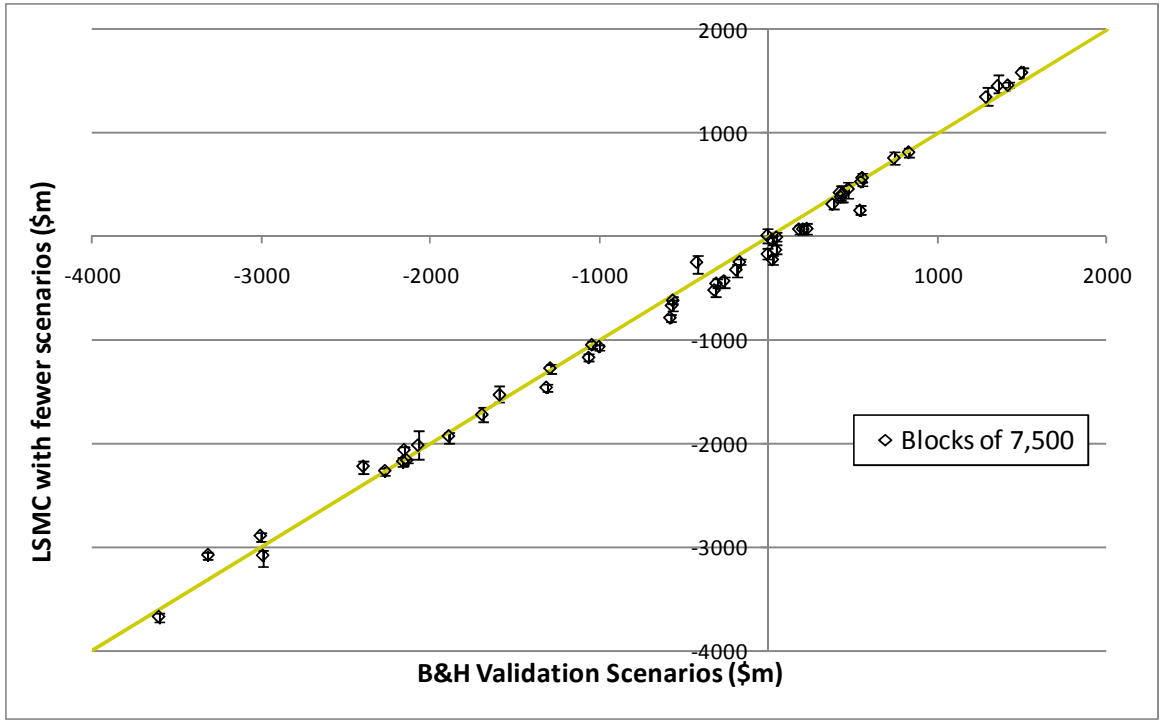
Validation points chosen to test function in different regions, or in regions of high sensitivity

LSMC with fewer Fitting Points

We have shown results for 25,000 LSMC fitting points

» How well does the method work with fewer points?

Validation for 7,500 points (15,000 scenarios)



Number fitting Points	Absolute Difference ($m)
4000	101
7500	94
10000	92
12500	90
25000	59

Advantages of LSMC

Speed: Can generate accurate results with fewer scenarios

» Much faster than other methods e.g. Brute Force or Curve Fitting

Accuracy

» Covers the entire risk spectrum that even a brute force approach with a traditional real-world scenario generator won't do.

» Accuracy can be readily validated.

Usage: Ease of analytics

» Stress-testing, and analysis of risk drivers can all be done in literally milli-seconds

Stability: Re-fitting only necessary if there is a large change in the liability profile

Adaptability

» Addition of non-market risk drivers into the function

» Multi-year projections

» Fit to variety of metrics

» Can be used for assets as well as liabilities - enables complex assets (e.g., structured products) to be modeled

Q&A