

# Proxy Modeling

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# Agenda

## 1. Introduction to Modeling Efficiency

## 2. Proxy Modeling

- Curve Fitting
- “Curve Fitting+”: Least Squares Monte Carlo

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# Introduction to Modeling Efficiency

# Examples of modeling efficiency

## » Modeling techniques

- Traditional actuarial model building (cell compression) – Places assets and liabilities into “like” categories/buckets, and these buckets then used for valuation
- Cluster modeling – More advanced form of cell compression
- Scenario reduction – Pick a subset of scenarios from a “full” population that still adequately captures all the risk exposures
- Replicating portfolio – Find a basket of assets that mirrors the liabilities and then use this basket of assets for valuation work (usually more manageable than working with liability models)
- Proxy modeling – Fit a function (proxy) to the assets and/or liabilities and use the function for valuation work. Proving to be very effective for “nested stochastic.”

## » Technology

- Grid computing
- Cloud computing
- GPGPU (General Purpose Graphics Processing Unit) processors

# Why is modeling efficiency becoming increasingly important?

## » **Small companies**

- Smaller companies likely not to have any VA business so up to this point likely to have had no requirement for stochastic modeling
- Stochastic modeling required under VM 20 is going to have a significant impact
- Some may already be up against systems issues just doing cash-flow testing

## » **Hedge effectiveness testing**

- In order to get credit for hedging in reserves/capital under AG43/C3 Phase II need to project hedges under a variety of stresses, and ideally do it on a fully stochastic basis. Hence a “nested stochastic” or “stochastic-on-stochastic” computation, which will be very onerous if done on a “brute force” basis.

## » **ORSA**

- Projection of stat or economic capital again potentially means a nested stochastic calculation

## » **Non-regulatory applications also create heavy computational demands**

- Internal EC and ALM
- Product pricing where stat reserves/capital need to be projected

# AAA MEWG

- » The regulators and LFS/RM Committee recognized that some of the calculations envisioned by a PBA approach to reserves and capital can be onerous
- » The purpose of the Modeling Efficiency Work Group (MEWG) is to provide information to the regulators and LFS/RM, and valuation actuaries on ways in which these calculations can be made more “efficient”
  - By efficient we mean ways in which modeling techniques and/or technology can be used to help ease the burden of huge processing requirements without affecting the accuracy of the results
  - For example, a scenario reduction technique is only “efficient” if it leads to the same result as a calculation using a larger scenario set
- » Our focus is on providing information and data from which interested parties can make informed decisions. We categorically do not attempt to favor any one tool or technique over another.
  - In the spirit of true PBA, and given some techniques may be more appropriate than others depending on the application/metric you are looking at, we believe prescribing anything in this area would be inappropriate.

## What has MEWG achieved?

- » Created bibliography of published materials regarding modeling efficiency
- » Two industry surveys:
  - “Modeling Efficiency Work Group Survey – November 2007”
    - » 29 participant companies.
    - » 80% using model building techniques; 40% using “scenario design”
    - » 25% using “hardware design;” 30% “software design”
  - “Usage of Modeling Efficiency Techniques in the US Life Insurance Industry” – April 2013
    - » 51 participants
    - » 25% using cell compression outside of traditional actuarial mapping
    - » 46% of participants now indicate using scenario reduction techniques (including the Academy scenario picking tool)
    - » Proxy approaches gaining popularity
- » MEWG provided input and guidance on drafting “modeling efficiency” sections of VM 20
- » MEWG members have also been very active in writing papers and presenting at conferences on modeling efficiency topics

## A new era for MEWG

- » MEWG presented at Aug 22 LATF meeting, and coming out of that meeting, regulators emphasized two key areas they need help with going forward:
  - Continued ad hoc advice and support from time-to-time, as issues arise, similar to what we did for VM 20
  - More pressing and difficult area: How do they assess any particular technique that is being used by a company they are reviewing?
- » Regulator assessment
  - Resources
    - » Do AAA working groups and/or the regulators need to engage quant (non-actuarial) professionals?
  - Understanding
    - » Glossary of terms
    - » What are key metrics for analysis?
    - » Update recent SOA survey of techniques

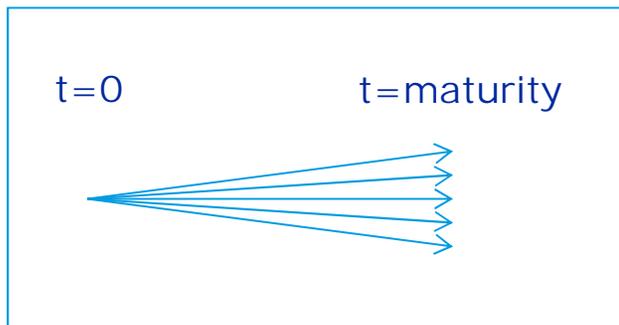
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## Proxy Modeling

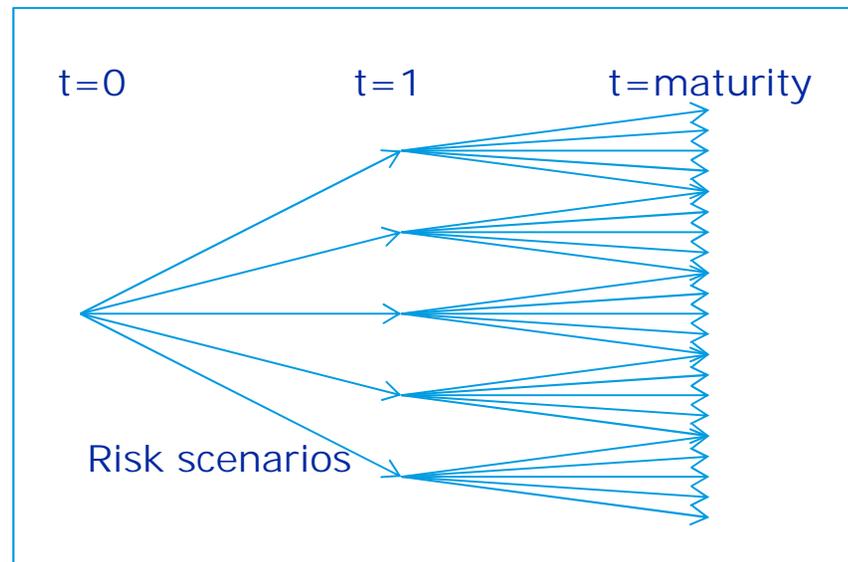
# The Nested Stochastic Problem

- » Complex liabilities such as VA living benefits may require a Monte Carlo simulation for valuation
- » This introduces a 'Nested Stochastic' problem for some applications, e.g., EC calculated on a 1-year VaR MC basis
  - So perhaps 1,000 outer scenarios x 1,000 inner scenarios = 1 million scenarios

t=0 valuation



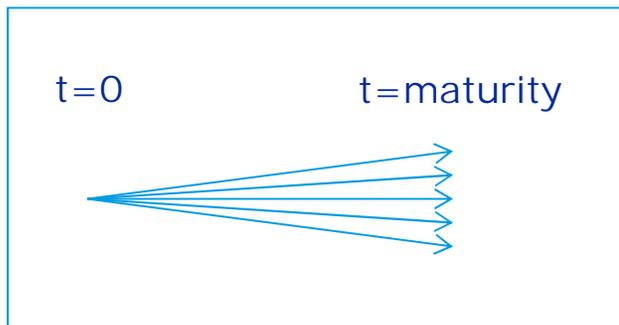
t=1 valuation projection



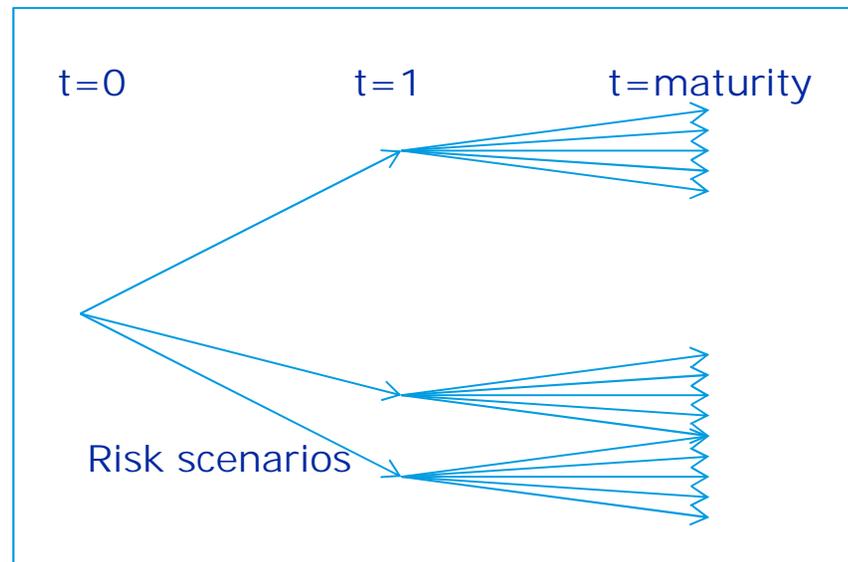
## Proxy Approach: Curve Fitting

- » Curve Fitting is a general technique to make Nested Stochastic problems feasible
- » Rather than run many Real-World or Risk Scenarios, we run a small number, then perform a regression of these values against a number of key risk drivers.
  - Produces a proxy function
  - May enable us to get our 1 million scenarios down to 100 outer x 1,000 inner = 100,000 scenarios
- » Issue: curve only good for the points that we've fitted to

t=0 valuation



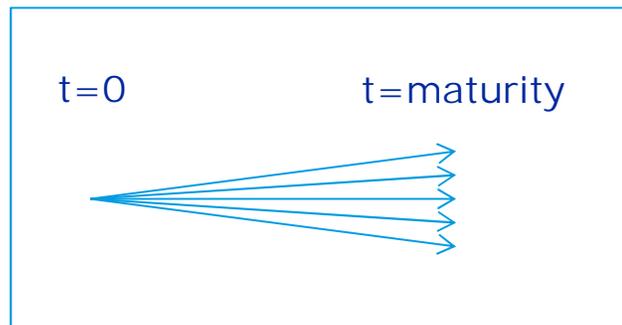
t=1 valuation projection



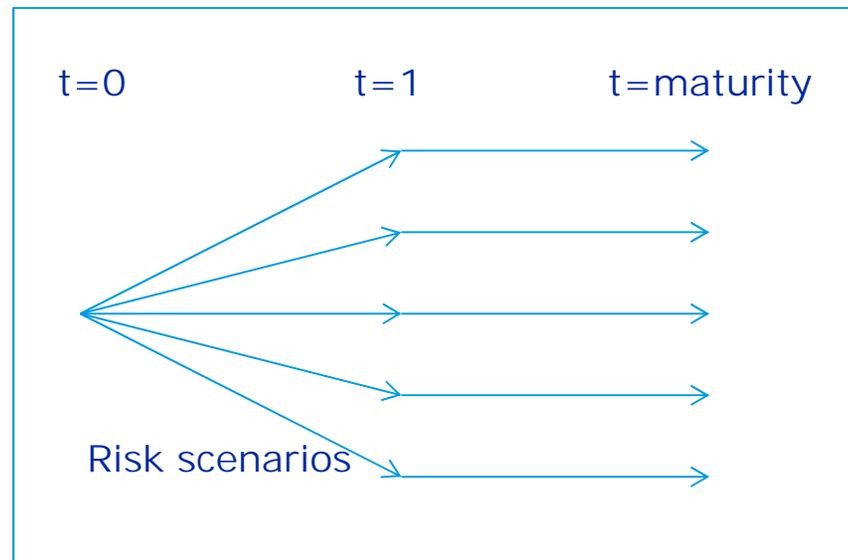
## Proxy Approach 2: Least Squares Monte Carlo (LSMC)

- » LSMC = “Curve Fitting+”
- » Rather than run many valuation scenarios for each Risk Scenario, we run a small number, then perform a regression of these values
  - Produces a proxy function
  - May enable us to get our 1 million scenarios down to 20,000 outer x 1 inner = 20,000 scenarios
  - Permits us to fit to many points, as opposed to the handful we have for Curve Fitting, hence we get a curve that works across the entire risk spectrum
  - It works because on average the errors offset each other

t=0 valuation



t=1 valuation projection

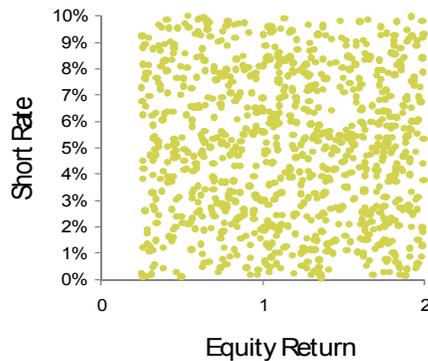


# Process for Proxy Generation using LSMC

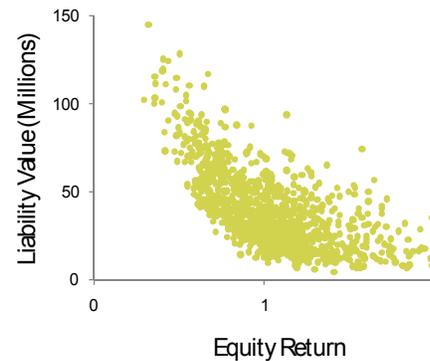
There are four main steps followed to derive the proxy function

Identify risks and generate 'fitting points'

Step 1



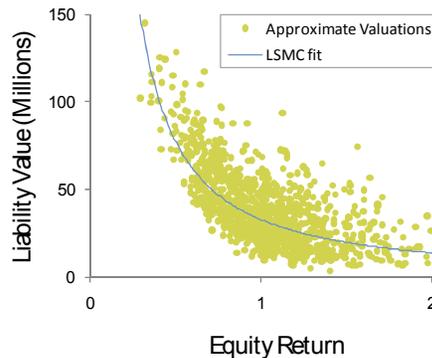
Step 2



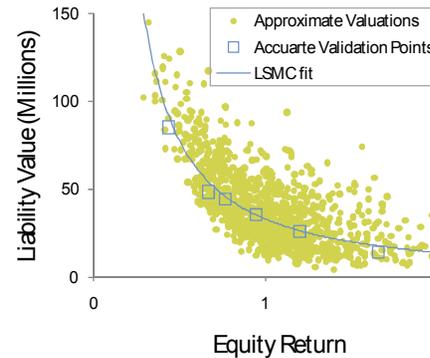
Calculate PV for each fitting point

Run optimization to fit PVs

Step 3

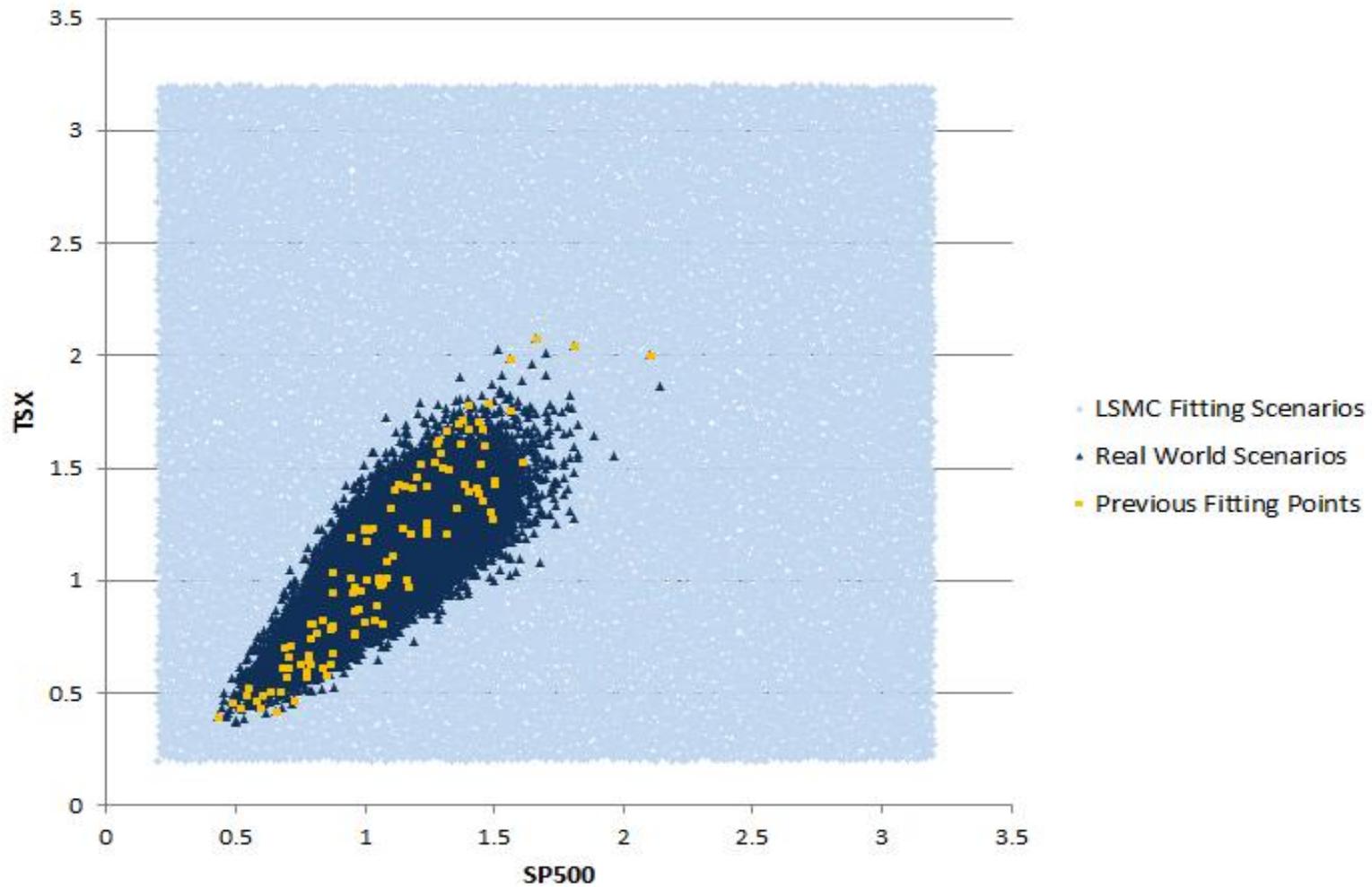


Step 4

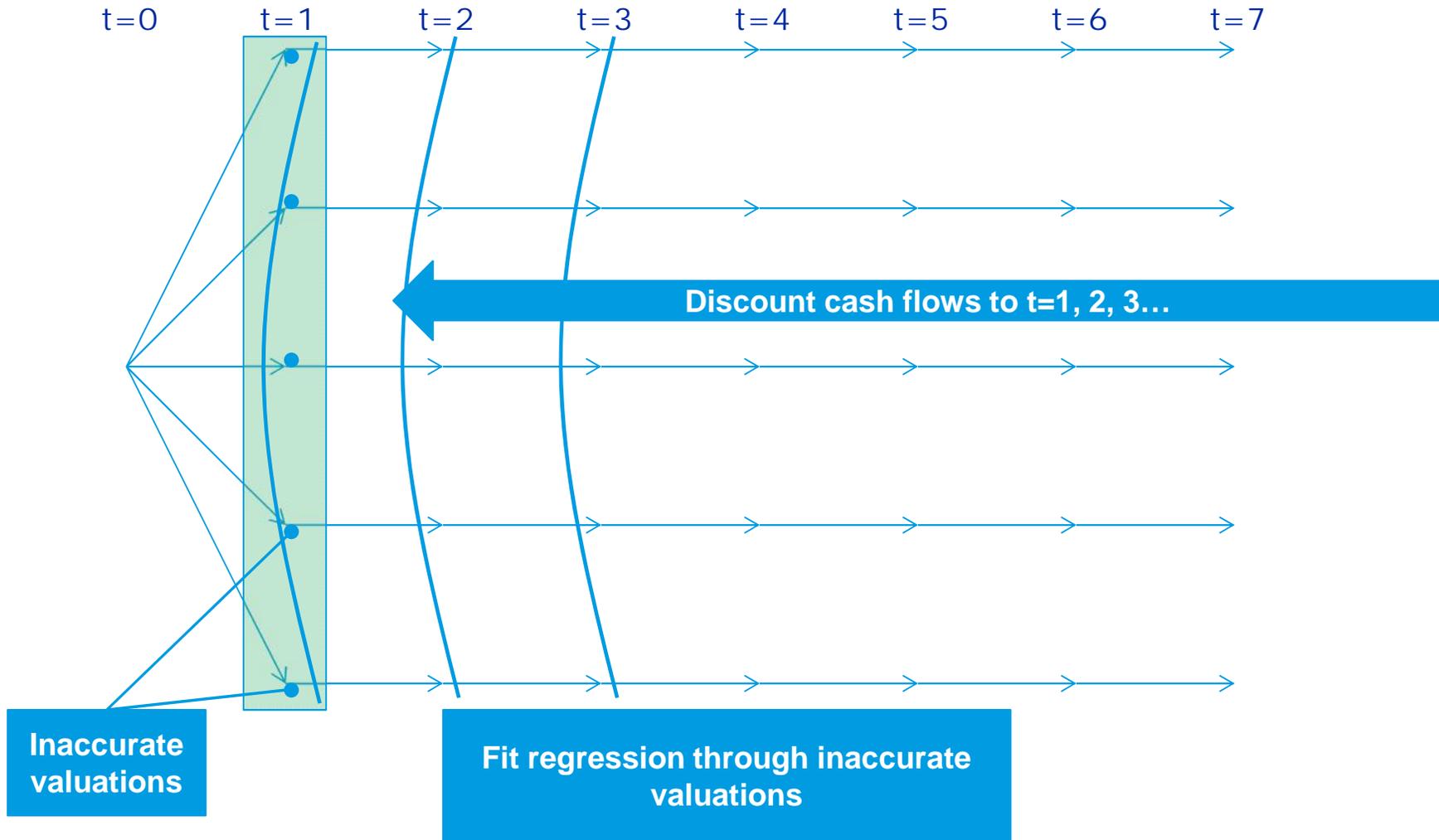


Validate proxy function using accurate valuations

# Model Comparison – LSMC actually more accurate than Brute Force



# Multi time-step LSMC – fitting scenarios



# LSMC Example: Variable Annuities

## Choosing risk drivers

Balance risk coverage with dimensionality of problem and risk budget

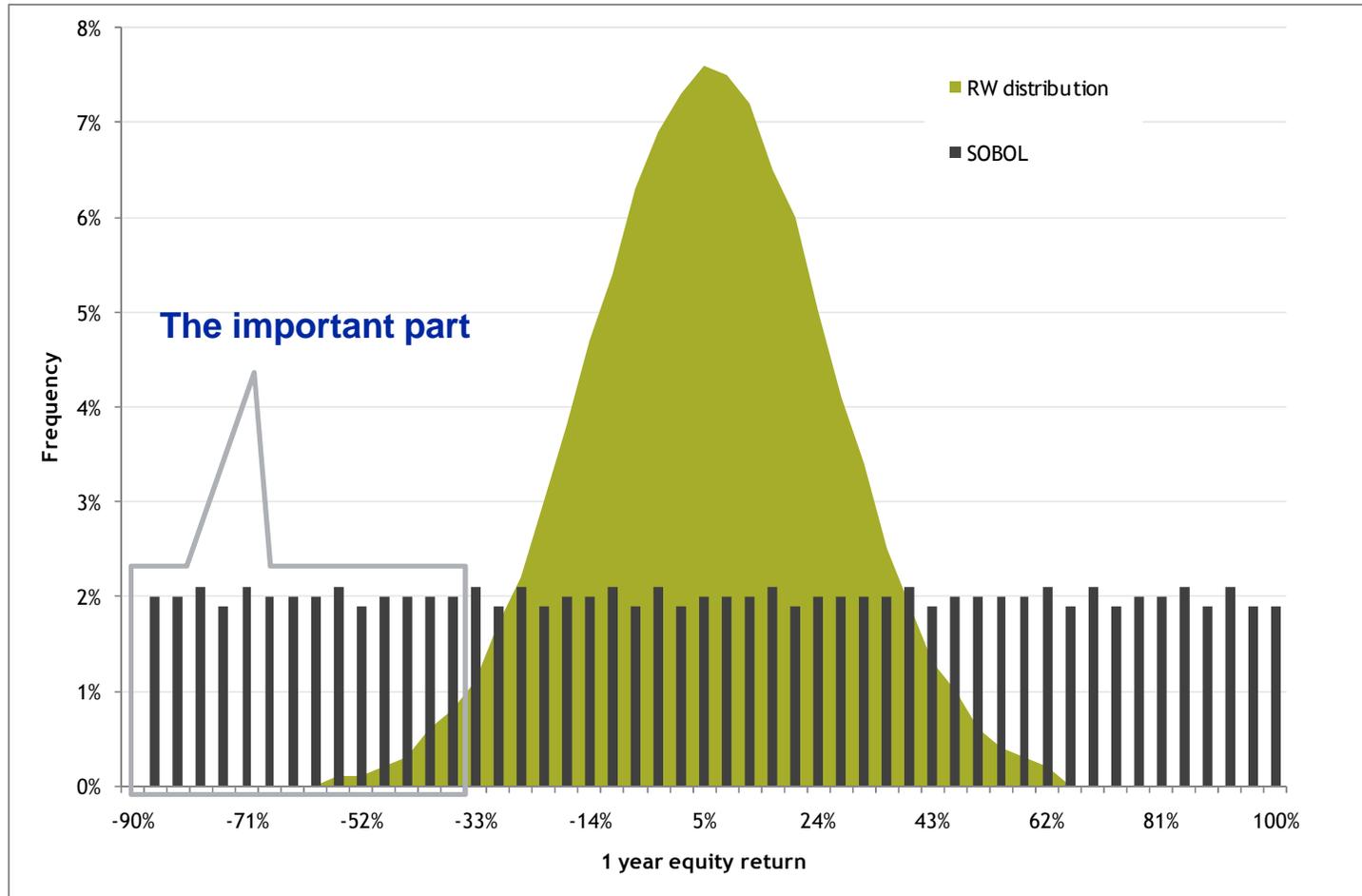
Example: VA block of business

- » **Treasury curve level:** parameterized by two principal components, PC1 and PC2.
- » **Equity level:** the return of the S&P500 index over 1 year
- » **Equity volatility:** the option implied volatility of the S&P500 in 1 year
- » **Swap spread:** the difference between the Treasury and Swap curves in 1 year

Key consideration: fully spanning risk driver space

- » Interested in the rare, extreme events
- » But Real World risk factor distributions rarely contain rare events, by definition
- » Instead we create specialist “fitting” scenarios

# Generating Fitting Scenarios



# Proxy Function

For the VA book we generated a polynomial with 47 terms:

» P[,] are Legendre Polynomials e.g. P[4,2] is 2<sup>nd</sup> order Legendre in 4<sup>th</sup> risk driver

Term	Coefficient	Term (cont)	Coefficient (cont)
(Intercept)	-992032984.7	P[4,1]:P[3,1]	-136432879.5
P[1,1]	761762317.8	P[4,2]:P[2,1]	151225297.4
P[4,1]	1825715949	P[4,1]:P[5,2]	-148168834.3
P[5,1]	-1395382145	P[4,1]:P[2,1]	-106150736.7
P[4,2]	-758349090.5	P[4,2]:P[3,1]	118719778.8
P[3,1]	573661632.5	P[1,1]:P[2,1]	-60328828.74
P[2,1]	550827457.8	P[1,1]:P[5,2]	50714322.09
P[5,2]	-241427978.9	P[2,1]:P[1,2]	-35340421.37
P[4,3]	149133865.5	P[4,1]:P[1,2]	-34754483.49
P[5,3]	75635590.08	P[5,1]:P[1,2]	-33940634.71
P[3,2]	-62050937.12	P[3,1]:P[5,2]	31296183.31
P[4,4]	56961766.44	P[3,1]:P[1,2]	-29170013.58
P[1,2]	-37964304.99	P[2,1]:P[5,2]	27967168.13
P[2,2]	-15170800.63	P[1,1]:P[2,2]	-38769610.03
P[4,5]	22140611.05	P[5,1]:P[2,2]	28575088.78
P[4,1]:P[5,1]	-696164796.1	P[1,1]:P[4,1]:P[5,1]	411342943.2
P[1,1]:P[5,1]	385012027.6	P[4,1]:P[5,1]:P[3,1]	236861499.7
P[1,1]:P[3,1]	-289866012.2	P[4,1]:P[5,1]:P[2,1]	265609661.1
P[5,1]:P[2,1]	284099296	P[1,1]:P[5,1]:P[2,1]	64154799.17
P[5,1]:P[3,1]	263166778.5	P[1,1]:P[4,1]:P[2,1]	-64631715.07
P[3,1]:P[2,1]	-209026930.8	P[1,1]:P[5,1]:P[3,1]	-60439171.12
P[1,1]:P[4,2]	203983067.4	P[5,1]:P[3,1]:P[2,1]	-37145207.65
P[5,1]:P[4,2]	192324722.4	P[1,1]:P[4,1]:P[3,1]	-33812469.43
P[1,1]:P[4,1]	-147402170		

Risk Driver	
1	PC1
2	PC2
3	Swap Spread
4	Equity Value
5	Equity Vol.

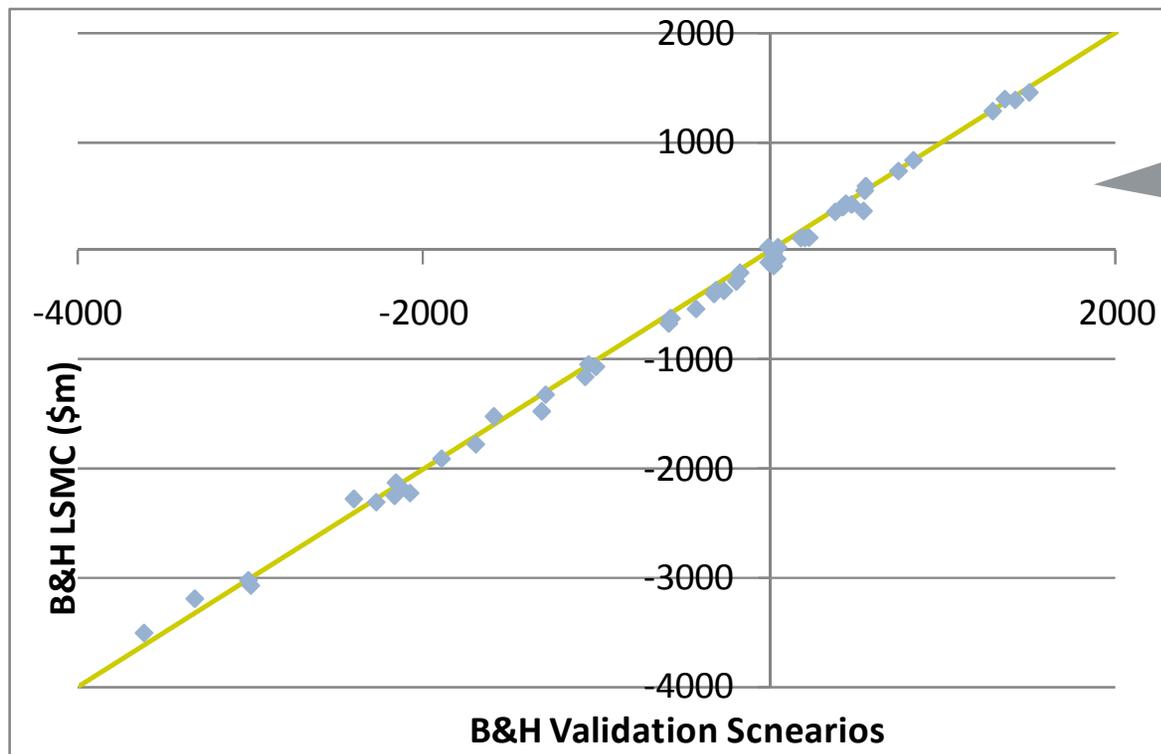
Up to 5<sup>th</sup> order in equity value term

Not all possible cross terms -> not overfitting

# Validation

Comparison of 1-year surplus calculated using the proxy and calculated using 1,000 RN scenarios

- » The average difference between the validation and the proxy function is \$60 million



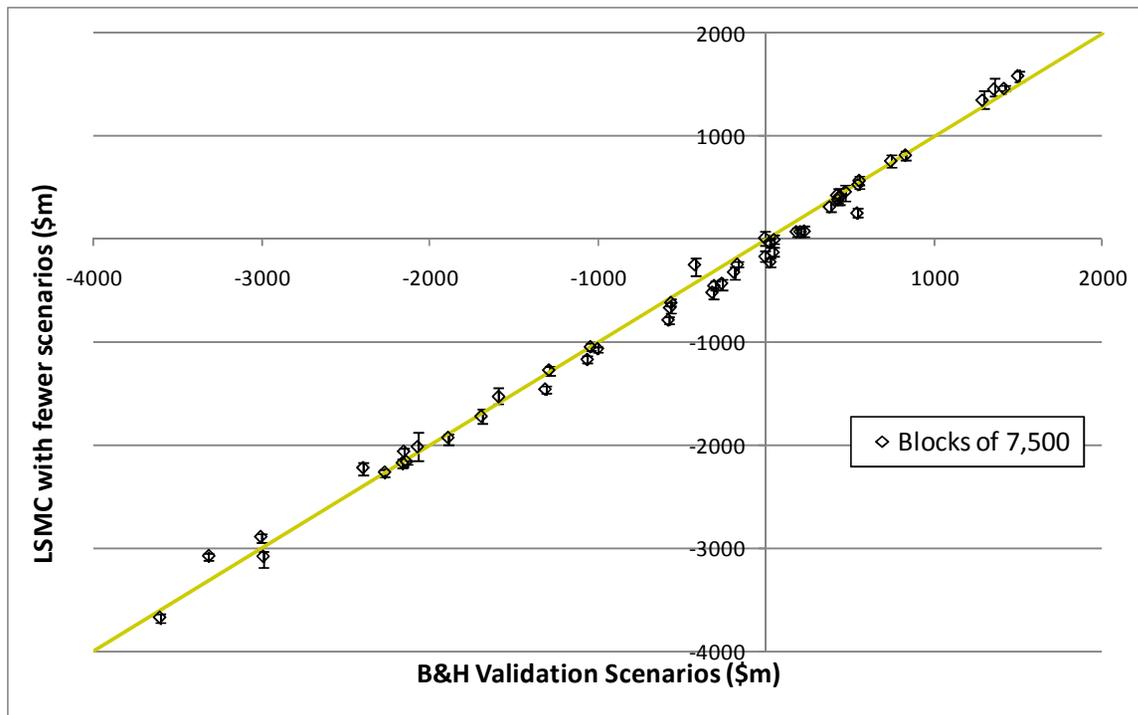
Validation points chosen to test function in different regions, or in regions of high sensitivity

# LSMC with fewer Fitting Points

We have shown results for 25,000 LSMC fitting points

- » How well does the method work with fewer points?

Validation for 7,500 points (15,000 scenarios)



Number fitting Points	Absolute Difference (\$m)
4000	101
7500	94
10000	92
12500	90
25000	59

## Advantages of LSMC

**Speed:** Can generate accurate results with fewer scenarios

- » Much faster than other methods e.g. Brute Force or Curve Fitting

### **Accuracy**

- » Covers the entire risk spectrum that even a brute force approach with a traditional real-world scenario generator won't do.
- » Accuracy can be readily validated.

**Usage:** Ease of analytics

- » Stress-testing, and analysis of risk drivers can all be done in literally milli-seconds

**Stability:** Re-fitting only necessary if there is a large change in the liability profile

### **Adaptability**

- » Addition of non-market risk drivers into the function
- » Multi-year projections
- » Fit to variety of metrics
- » Can be used for assets as well as liabilities - enables complex assets (e.g., structured products) to be modeled

# Q&A

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